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INFLUENCE OF THE ELECTRON-PHONON COUPLING ON SUPERCONDUCTING CONTACT AND THE PROPERTIES OF SUPERCONDUCTOR-SEMIMETAL (OR SEMICONDUCTOR) SYSTEMS

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ABSTRACT

Consideration of the proximity effect, electron-phonon coupling and size quantization results in a peculiar behavior of the critical temperature T_c of a contact, containing superconducting and semimetal (or semiconductor) thin films. The method of thermodynamic Green function is applied. The dependence of T_c on the thickness of the barrier is discussed. The size quantization leads to additional oscillations of T_c and to the possibility of observing peculiar charge density waves. The experimental data are discussed.

I. INTRODUCTION

It is well-known that systems containing a contact of superconducting and normal films possess very interesting physical properties.

In this paper we consider the situation when the normal film is a semimetal (SM) or semiconductor (SC). SM and SC films are characterized by a number of peculiarities which allow one to observe experimentally some interesting phenomena.

Systems containing thin superconducting and not-metallic films have been studied by several experimental groups.[1-4] The dependence of the critical temperature T_c on the thickness of the non-metallic film was observed. Usually, this dependence is non-monotonic.

The properties of superconductor-semimetal (S-SM) and superconductor-semiconductor (S-SC) systems depend on a number of different factors. That is why the values of different parameters and, first of all, T_c can vary very noticeably. We consider here the influence of several main factors:

1. The proximity effect. The Cooper pairs can move into the non-metallic film in the presence of a good electric contact, and this results in the appearance of an induced superconducting state of the SM or SC film. The back flow of electrons also exists.
2. The size quantization (SQ) of the transverse motion. The phenomenon is characterized by oscillations of the density of states. The best conditions for this effect are realized in

thin semimetal and semiconductor films (see below), that is, in the presently considered case.

3. The change of the phonon spectrum caused by non-metallic covering. This change also affects the critical temperature. Note that the peculiarities of the dispersion relation lead to the appearance of specific charge density waves.

We take into account all the mentioned factors. We focus on the problem of calculation of T_c. The effect of the proximity and the size quantization on other properties of the considered system will be examined in the framework of the phonon model elsewhere.

The plan of the present paper is as follows. Section II addresses the problem of obtaining the main equation. We use the method of thermodynamic Green functions and take into account the electron-phonon interaction directly. We consider the effect of covering on T_c in Section III. The proximity effect, the size quantization and specific charge density waves are discussed in Sections IV and V. In Sec. VI we examine the case when the effective constants of both films are not equal to zero. Note, that the results of Sections II, IV, VI can also be applied to the situation when both films are metallic.

II. MAIN EQUATIONS

Consider a system containing superconducting (α) and normal (β) thin films. We shall consider in more detail (see below, Section V) the case, when β film is a semimetal (or a semiconductor).

Denote by T_c^{α} the critical temperature of the isolated superconducting film and suppose the thicknesses L_α and L_β satisfy the conditions $L_\alpha \ll \xi$, $L_\beta \ll \xi$, where ξ is the coherence length. Moreover, it is supposed that the metallic film is "dirty" in the Anderson sense.[5] The size quantization has been observed experimentally by investigation of thin films in the region 10 Å $\lesssim L_\beta \lesssim 10^3$ Å[6-9] (see, e.g., the excellent review[10]). Hence, the mentioned conditions are perfectly realistic.

Under these conditions we can use the McMillan model of the proximity effect.[11] The electron-phonon interaction considered explicitly was included in the McMillan model in Ref. [12] for the purpose of calculating the energy gap function.

To calculate the critical temperature of S-SM or S-SC system it is very convenient to use the method of thermodynamic Green function (see, e.g. Refs. [13,14]. Let us introduce the self-energy parts $\Sigma_2{}^{\alpha}$ and $\Sigma_2{}^{\beta}$ describing the pairing in the "α" and "β" films. The equations for the self-energy parts are (Fig. 1)

Fig. 1

or, in the analytical form:

$$\Sigma_2^\alpha = \Sigma_{2,ph}^\alpha + \tilde{T}^2 \int d\vec{p}' \, F^\beta(\vec{p}',\omega_n) \tag{1}$$

$$\Sigma_\alpha^\beta = \tilde{T}^2 \int d\vec{p}' \, F^\alpha(\vec{p}',\omega_n) \tag{2}$$

$$\Sigma_{2ph} = T \sum_{\omega_{n'}} \int d\vec{p}' \, g_\alpha^2 (\vec{p},\vec{p}') \, D(\omega_n - \omega_{n'}, \omega(q)) \, F^\alpha(\vec{p}',\omega_{n'}) \tag{3}$$

$D = \omega^2(q)/[\omega^2(q) + (\omega_n - \omega_{n'})^2]$ is the phonon Green function, $\omega_n = (2n+1)\pi T$, $g_\alpha{}^2(\vec{p},\vec{p}')$ describes the electron-phonon interaction, F^α and F^β are anomalous Green functions[15-17]

$$F^\alpha(\omega_n,\vec{p}) = - \Sigma_2^\alpha (\omega_n,\vec{p})/[\omega_n^2 Z_\alpha^2 + \xi_\alpha^2(\vec{p}) + \Sigma_2^{(\alpha)^2}(\omega_n,\vec{p})]$$

$$F^\beta_{(\omega_n \, p)} = - \Sigma_2^\beta (\omega_n,\vec{p})/[\omega_n^2 Z_\beta^2 + \xi_\beta^2(\vec{p}) + \Sigma_2^{(\beta)^2}(\omega_n,\vec{p})] \tag{4}$$

Here $\xi_{\alpha;\beta}$ is the energy of an ordinary electron referred to the Fermi level, $Z_{\alpha;\beta}$ is the renormalized parameter, $\tilde{T}$ is the tunneling matrix element.[18] The Coulomb pseudopotential μ^* can be included in the usual way.

Equations (1)-(3) are written for the case when $g_\beta = 0$. Hence, the existence of pair condensate in the β film is due to the proximity effect only. The case $g_\beta \neq 0$ is considered below (see Sec. VI).

The electron-phonon interaction is taken into account directly. We restrict ourselves to the consideration of the phonon model with weak coupling (see, e.g., Refs. [19,20]. The consideration of the strong coupling is straightforward and will be given elsewhere.

The renormalized parameters are connected (in the weak electron-phonon coupling approximation) with the proximity effect only and they are equal to (see Ref. [11]):

$$Z_\alpha(\omega_n) = 1 + \Gamma^{\alpha\beta}/|\omega_n| \tag{4'}$$

$$Z_\beta(\omega_n) = 1 + \Gamma^{\beta\alpha}/|\omega_n| \tag{5}$$

Here

$$\begin{aligned} \Gamma^{\alpha\beta} &= \pi \tilde{T}^2 \nu_\beta V_\beta \\ \Gamma^{\beta\alpha} &= \pi \tilde{T}^2 \nu_\alpha V_\alpha \end{aligned} \tag{6}$$

where ν_α and ν_β are the densities of states (per unit of volume). The quantity $\Gamma^{\beta\alpha}$ can be written in the form (see Ref. [11]):

$$\Gamma^{\beta\alpha} = V_{F_\perp} \sigma/2BL_\beta \tag{6'}$$

where $V_{F_\perp}$ is the Fermi velocity, σ is the barrier penetration probability, and B is a function of the ratio of the mean free path to the film thickness.

Our goal is to calculate T_c. T_c can be evaluated from Eqs. (1)-(4). If $T = T_c$, we should put $\Sigma^\alpha = 0$ and $\Sigma^\beta = 0$ in the denominators of (4). Introduce the function $C^\alpha(\omega_n)|_{T_c} = (\Sigma_2(\omega_n)|_{T_c})/(1-T/T_c)^{1/2}$. According to Eqs. (1)-(4), we obtain:

$$C^\alpha(\omega_n) = C^\alpha_{2,ph}(\omega_n) + \Gamma_{\alpha\beta} \frac{C^\beta(\omega_n)}{\left|\omega_n Z^\beta(\omega_n)\right|} \tag{7}$$

$$C^\beta = \Gamma_{\beta\alpha} \frac{C^\alpha(\omega_n)}{\left|\omega_n Z^\alpha(\omega_n)\right|} \tag{8}$$

The term $C^\alpha{}_{ph}(\omega_n)$ is described by Eq. (3) and can be written in the form:

$$C^\alpha_{ph}(\omega_n) = \pi T \sum_{\omega_{n'}} \int d\omega\, g_\alpha(\omega) \frac{\omega^2}{\omega^2+(\omega_n-\omega_{n'})^2} \times \frac{C^\alpha(\omega_{n'})}{\left|\omega_{n'}\right| Z^\alpha(\omega_{n'})\big|_{T_c}} \tag{9}$$

We transformed to integration over frequencies and introduced the function (see, e.g., Refs. [19,20].

$$g(\omega) = \frac{\zeta}{p_F^2} \sum_j \frac{qdq}{d\omega} \frac{s^2q^2}{\omega^2} \gamma_j \tag{10}$$

where ζ is the Fröhlich parameter, q is the phonon momentum, s is the velocity of sound, p_F is the Fermi momentum, $\gamma_j(q) \sim 1$, and j is the order number of the phonon branch.

The function $g(\omega)$ can be written in the well-known form:[21] $g(\omega) = a(\omega)F(\omega)$, where $F(\omega)$ describes the electron-phonon interaction. This function can be found from tunnel measurements.[22]

Based on Eqs. (7) and (8), one can express C^{α} in terms of $C_{ph}{}^{\alpha}$.

$$C^{\alpha}(\omega_n) = C^{\alpha}_{ph} \frac{(1+\Gamma_{\alpha\beta}/\omega_n)(1+\Gamma_{\beta\alpha}/\omega_n)}{1 + (\Gamma_{\alpha\beta}+\Gamma_{\beta\alpha})/\omega_n} \tag{11}$$

Using this expression and Eq. (9) we arrive at the following equation:

$$1 = 2\pi T \sum_{\omega_{n'}>0} \int d\omega g_{\alpha}(\omega) \frac{\omega^2}{\omega^2+\omega_{n'}} \cdot \frac{1}{\omega_{n'}} \cdot \frac{1+\Gamma_{\beta\alpha}/\omega_{n'}}{1+(\Gamma_{\alpha\beta}+\Gamma_{\beta\alpha})/\omega_{n'}}\Bigg|_{T_c} \tag{12}$$

We took advantage of the weak coupling approximation. If one calculates T_c in this approximation, one can neglect the term ω_n in the denominator of the integrand in Eqs. (3), (9). As was shown by the author in[19], this approximation is valid to within small corrections in the coefficient before the exponent in the expression describing T_c. This approximation allows us to neglect the dependence of C^{α} on ω_n and we arrive at Eq. (12).

Equation (12) allows us to calculate the critical temperature. It can be rewritten in the form:

$$\begin{aligned} 1 = {} & \int d\omega\, g_{\alpha}(\omega)\, 2\pi T \sum_{\omega_n>0} \frac{\omega^2}{(\omega^2+\omega_n^2)\omega_n} \\ & - \Gamma_{\alpha\beta} \int d\omega\, g_{\alpha}(\omega)\, 2\pi T \sum_{\omega_n>0} \frac{\omega^2}{(\omega^2+\omega_n^2)\omega_n} \cdot \frac{1}{\omega_n+\Gamma} \end{aligned} \tag{13}$$

The sums on the right-hand side of Eq. (13) can be evaluated and we obtain

$$1 = I_1 + I_2 \qquad (14)$$

$$I_1 = \int d\omega\, g_\alpha(\omega) \ln \frac{2\omega\gamma}{\pi T_c} \qquad (15)$$

$$I_2 = -\frac{\Gamma_{\alpha\beta}}{\Gamma} \int d\omega\, g_\alpha(\omega) \left\{ \left[\psi\left(\frac{1}{2} + \frac{\Gamma}{2\pi T_c}\right) - \psi\left(\frac{1}{2}\right)\right] \right.$$

$$\left. \times \frac{\omega^2}{\omega^2+\Gamma^2} + \frac{\Gamma^2}{\omega^2+\Gamma^2} \ln \frac{2\omega\gamma}{\pi T_c} \right\} \qquad (16)$$

where $\Gamma = \Gamma_{\alpha\beta} + \Gamma_{\beta\alpha}$. We neglect the small term $[-\pi\omega\Gamma/(2(\omega^2+\Gamma^2))$ in the brackets in Eq. (16); $\gamma \approx 1.78$.

We see that the value of T_c depends on the function $g_\alpha(\omega)$ and on the term I_2 describing the proximity effect. According to Eqs. (6) and (16) the term I_2 depends on the density of states ν^β.

We shall consider (see below, Sec. VI) the case when β film is semimetal or semiconductor. The interesting features of S-SM and S-SC systems are connected with the peculiarities of the density of states in these films.

It is worth noting that the function $g_\alpha(\omega)$ differs from the function $g_\alpha{}^o(\omega)$ describing the phonon spectrum and the electron-phonon interaction in an isolated metallic film. We consider thin metallic films and they are very sensitive to the covering in the sense of the change of $g_\alpha{}^o(\omega)$.

The function $g_\alpha(\omega)$ can be written in the form:

$$g_\alpha(\omega) = g_\alpha^o(\omega) + g_\alpha^{(1)}(\omega) \quad (17)$$

where $g_\alpha^{(1)}(\omega)$ is due to the influence of the covering. Denote by $T_c^\alpha \equiv T_c(L_\beta = 0)$ the critical temperature of an isolated α film. T_c^α is satisfied by the equation:

$$1 = \int d\omega\, g_\alpha^o(\omega) \ln \frac{2\omega\gamma}{\pi T_c^\alpha} \quad (18)$$

Using (18), we can reduce Eq. (14) to the form:

$$\begin{aligned} \ln \frac{T_c}{T_c^\alpha} &= \lambda_\alpha^{-1} \int d\omega\, g_\alpha^{(1)}(\omega) \ln \frac{2\omega\gamma}{\pi T_c^\alpha} \\ &- \frac{\Gamma_{\alpha\beta}}{\Gamma} \lambda_\alpha^{-1} \int d\omega\, g_\alpha(\omega) \left\{ \left[\psi\left(\frac{1}{2} + \frac{\Gamma}{2\pi T_c}\right) - \psi\left(\frac{1}{2}\right) \right] \right. \\ &\left. \times \frac{\omega^2}{\omega^2+\Gamma^2} + \frac{\Gamma^2}{\omega^2+\Gamma^2} \ln \frac{2\omega\gamma}{\pi T_c} \right\} \end{aligned} \quad (19)$$

where

$$\lambda_\alpha = \int g_\alpha(\omega)\, d\omega \quad (20)$$

Equations (14)-(16) (or Eq. (19)) are the basic equations of the theory. The first and second terms on the right-hand side of Eq. (19) describe the influence of different factors on T_c.

III. THE CHANGE OF THE PHONON SPECTRUM

As is well known, covering results in a distortion of the initial phonon spectrum, in the appearance of new modes and so on. It is particularly important in the investigation of thin superconducting films.

The influence of covering can be considered by analogy with the author's paper[23] on the change in T_c in superconductors which contain complex molecules.

The function $g_\alpha(\omega)$ can be written in the form Eq. (17). Suppose $g_\alpha^{(1)} > 0$, that is, the covering leads to an additional attraction between electrons. The critical temperature T_c^α of the isolated α film satisfies Eq. (18). Denote by $\Delta T_c \equiv T_c' - T_c^\alpha$ the change of the critical temperature caused by the term $g_\alpha^{(1)}(\omega)$. The quantity T_c', which is equal to the value of the critical temperature in the absence of the proximity effect, satisfies the equation (see Eq. (14), (15)):

$$1 = \int d\omega \, g_\alpha(\omega) \ln \frac{2\omega\gamma}{\pi T_c'} \quad . \tag{21}$$

According to Eq. (18) and (21), we obtain

$$\ln \frac{T_c'}{T_c^\alpha} = \frac{\lambda_1}{\lambda_\alpha} \ln \frac{2\langle\omega\rangle^{(1)}\gamma}{\pi T_c^\alpha} \tag{22}$$

where $\langle\omega\rangle^{(1)}$ denotes the mean value in the sense

$$\frac{\langle\omega\rangle^{(1)}}{T_c^\alpha} = \frac{1}{\lambda_1}\int d\omega\, g_\alpha^{(1)}(\omega)\, \ln\frac{\omega}{T_c^\alpha} \quad ;$$

$$\lambda_1 = \int d\omega g_\alpha^{(1)}(\omega) \quad ;$$

and λ_α is defined by Eq. (20). According to Eq. (22), we obtain

$$T_c' = T_c^\alpha\,(2\langle\omega\rangle^{(1)}\gamma/\pi T_c^\alpha)^{\lambda_1/\lambda_\alpha} \tag{23}$$

Usually $\lambda_1 \ll \lambda_\alpha$. We find then

$$\frac{\Delta T_c}{T_c^\alpha} = \frac{\lambda_1}{\lambda_\alpha}\ln\frac{2\langle\omega\rangle^{(1)}\gamma}{\pi T_c^\alpha} \tag{24}$$

If the function $g_\alpha^{(1)}(\omega)$ corresponds to the appearance of an additional peak at $\omega \simeq \omega_1$, then $\langle\omega\rangle^{(1)} \simeq \omega_1$ and we obtain

$$\frac{\Delta T_c}{T_c^\alpha} \simeq \frac{\lambda_1}{\lambda_\alpha}\ln\frac{2\omega_1\gamma}{\pi T_c^\alpha} \tag{25}$$

The change of T_c can be noticeable because of the presence of large logarithmic factors. If for example, $\lambda_1/\lambda \approx 0.1$ and $T_c{}^\alpha/\omega_1 \approx 0.1$ then $\Delta T_c/T_c{}^\alpha \approx 25\%$. Even if $\lambda_1/\lambda \approx 10^{-2}$ and $T_c{}^\alpha/\omega_1 \approx 10^{-2}$, then the increase of T_c can reach several percent.

Equations (24), (25) relate $\Delta T_c{}'$ to measureable quantities. The quantities λ_α, λ_1 can be obtained from tunnel measurements.

If the covering is characterized by the existence of soft modes, this leads to an increase of λ_1 and T_c. Size quantization results in the appearance of on additional softening mechanism (see below, Sec. V).

The increase of T_c as a function of L_β relative to $T_c{}^\alpha$ is observed often experimentally in the region of small $L_\beta \sim a$[1-4] (a is atomic distance) by the investigation of the systems Tl-Ge,[1] Mo-C, Te-C, V-C,[2] Al-SiO,[3], Pb-Ge and Pb-Si.[4] This increase is due to the considered mechanism's role in the region of small L_β. Note that this increase of T_c cannot be coupled with the term I_2 in Eq. (14), because the proximity effect tends to diminish the value of T_c (see below).

IV. PROXIMITY EFFECT

1. Critical temperature

Let us consider again the basic Eqs. (14-16). Equation (13) can be rewritten in the form:

$$\ln\frac{T_c}{T_c'} = -\frac{\Gamma_{\alpha\beta}}{\Gamma}\cdot\frac{1}{\lambda_\alpha}\int d\omega\, g(\omega)\left\{\left[\psi\left(\frac{1}{2}+\frac{\Gamma}{2\pi T_c}\right)-\psi\left(\frac{1}{2}\right)\right]\frac{\omega^2}{\omega^2+\Gamma^2}+\frac{\Gamma^2}{\omega^2+\Gamma^2}\ln\frac{2\omega\gamma}{\pi T_c}\right\} \tag{26}$$

Here T_c' is the solution of Eq. (19) and corresponds to the value of T_c in the absence of the proximity effect. It is worth noting that, generally speaking, $T_c' \neq T_c^\alpha$ (see Sec. III). The equality $T_c' = T_c^\alpha$ is valid if it is possible to neglect the change of the phonon spectrum.

Consider the case $\Gamma \gg T_c$. Using Eq. (26) and the asymptotic expression of the di-gamma function ($\psi(z) \simeq \ln z$), we obtain

$$\psi\left(\frac{1}{2}+\frac{\Gamma}{2\pi T_c}\right) - \psi\left(\frac{1}{2}\right) \simeq \ln\frac{2\Gamma\gamma}{\pi T_c} , \tag{27}$$

Hence, we arrive at the following equation:

$$\ln\frac{T_c}{T_c'} = -\frac{\Gamma_{\alpha\beta}}{\Gamma}\ln\frac{2\langle u(\omega)\rangle\gamma}{\pi T_c} \tag{28}$$

where the mean value is to be understood in the sense

$$\ln\frac{\langle u(\omega)\rangle}{T_c} = \frac{1}{\lambda_\alpha}\int d\omega\, g_\alpha(\omega)\ln\frac{u(\omega)}{T_c} \tag{29}$$

and

$$u(\omega) = \Gamma^{1-\delta}\omega^{\delta} \quad ; \quad \delta = \frac{\Gamma^2}{\omega^2+\Gamma^2} \tag{30}$$

After simple calculations we finally obtain

$$T_c = T_c' \left(\frac{\pi T_c'}{2\langle u\rangle\gamma}\right)^{\rho} \tag{31}$$

where

$$\rho = \Gamma_{\alpha\beta}/\Gamma_{\beta\alpha} \quad ; \quad \Gamma = \Gamma_{\alpha\beta} + \Gamma_{\beta\alpha} = \Gamma_{\beta\alpha}(1+\rho) \tag{32}$$

and T_c' and $\langle u\rangle$ are defined by Eqs. (19) and (29). Equation (31) is valid for an arbitrary ratio of $\langle\omega\rangle$ and Γ.

According to Eq. (6), we obtain

$$\rho = (\nu_\beta/\nu_\alpha)\ (L_\beta/L_\alpha) \tag{32'}$$

Note that if films "α" and "β" are ordinary metallic films, then (in the effective mass approximation)

$$\nu_\alpha = m_\alpha p_F^\alpha/\pi^2 \quad ; \quad \nu_\beta = m_\beta p_F^\beta/\pi^2 \tag{33}$$

and hence,

$$\rho = (m_\beta p_F^\beta/m_\alpha p_F^\alpha)\ (L_\beta/L_\alpha) \tag{34}$$

Here m_α, m_β are the effective masses and $p_F{}^\alpha$, $p_F{}^\beta$ are the Fermi momenta. If film "β" is a semimetal or degenerate semiconductor and is not size quantizing, Eqs. (31)-(34) are also valid. In this connection $p_F{}^\beta \sim n^{1/3}$, where n is the electron concentration. Then α and T_c can vary as functions of n. The special case of size quantization will be considered below (Sec. V).

It is very convenient to present the dependence Eq. (30) in the form

$$u = \Gamma\left(\frac{\omega}{\Gamma}\right)^{\delta} \tag{35}$$

or in the following dimensionless form:

$$y = t^{(1/1+t^2)} \tag{36}$$

where $y = u/\Gamma$; $t = \omega/\Gamma$. The function $y(t)$ is shown on Fig. 2. We see that if t is small ($\omega \ll \Gamma$), $y \approx t$ and $u \approx \omega$. In the opposite case of large $t(\omega \gg \Gamma)$ the quantity $u \to 1$ and $\lambda \approx \Gamma$.

Consider Eq. (31) and examine the limiting cases. Equation (29) can be written in the form

$$\ln\frac{\langle u\rangle}{T_c} = K_1 \ln\frac{\Gamma}{T_c} + K_2 \ln\frac{\langle\omega\rangle}{T_c} \tag{37}$$

where $$K_1 = \frac{1}{\lambda_\alpha}\int d\omega\, g(\omega)\, \omega^2/(\omega^2+\Gamma^2)$$

$$K_2 = \frac{1}{\lambda_\alpha}\int d\omega\, g(\omega)\, \frac{\Gamma^2}{\omega^2+\Gamma^2}\, \ln\left(\frac{\omega}{T_c}\right)/\ln\frac{\langle\omega\rangle}{T_c} \tag{37'}$$

Suppose that $\Gamma \ll \langle\omega\rangle$, or, more exactly, $K_2 \ll 1$. Then $K_1 \approx 1$ and, according to Eq. (37), $\langle u\rangle \approx \Gamma$. Then we arrive at the following equation:

$$T_c = T'_c\left(\frac{\pi T'_c}{2\Gamma\gamma}\right)^{\rho} \tag{38}$$

where p is defined by Eq. (32).

If the change of phonon spectrum is small, then $T_c' = T_c^{\alpha}$ and we obtain

$$T_c = T_c^{\alpha}\left(\frac{\pi T_c^{\alpha}}{2\Gamma\gamma}\right)^{\rho} \quad . \tag{38'}$$

This expression was obtained by McMillan.[11] If Γ is large ($\Gamma \gg \langle\omega\rangle$, or, more precisely, $K_1 \ll 1$), then $K_2 \approx 1$ and $\langle u\rangle \approx \langle\omega\rangle$. Naturally, these estimates are consistent with the estimates of the function y(t) (see above). Then we arrive at the expression (if $T_c' = T_c^{\alpha}$), which corresponds to Cooper's case:[11,24]

$$T_c = T_c^{\alpha}\left(\frac{\pi T_c^{\alpha}}{2\langle\omega\rangle\gamma}\right)^{\rho} . \tag{39}$$

In the general case one should use Eqs. (28)-(31) which are valid for an arbitrary relation between Γ and $\langle\omega\rangle$.

If $\Gamma \lesssim T_c$, $\langle\omega\rangle \gg T_c$, Eq. (31) is not applicable. According to Eq. (26), we obtain in this case

$$\ln\,(T_c/T_c') = -F(\rho,\Gamma/T_c) \quad , \tag{40}$$

where

$$F(\rho,\Gamma/T_c) = (\Gamma_{\alpha\beta}/\Gamma)\,\left[\psi\left(\frac{1}{2} + \frac{\Gamma}{2\pi T_c}\right) - \psi(\tfrac{1}{2})\right]$$

$$\rho = \Gamma_{\alpha\beta}/\Gamma_{\beta\alpha}$$

The function F does not depend on the phonon spectrum directly. Eq. (40) was considered in Ref. [11] (in the case, when $T_c' = T_c^{\alpha}$). Note, that if $\rho \ll 1$, we obtain

$$T_c = T_c' \exp\{-F(\rho, \Gamma/T_c')\} \tag{41}$$

According to Eq. (31), T_c strongly depends on the quantity $\langle u \rangle$. The value of $\langle u \rangle$ (see Eq. (29), (30)) is related to Γ and to the function $g(\omega)$, or, roughly speaking, the value of $\langle u \rangle$ depends on the relation between Γ and $\langle \omega \rangle$. Γ and $\langle \omega \rangle$ can be varied very noticeably and this results in the corresponding change of T_c.

2. <u>Dependence $T_c(d)$</u>

The peculiar behavior of $\langle u \rangle$ (see Fig. 2) allows us to propose the following experiment. The parameter $\Gamma = \Gamma_{\alpha\beta} + \Gamma_{\beta\alpha}$ contains (see Eq. (6)) the tunneling matrix element and its value depends on the thickness of the barrier, e.g., on the thickness of oxide d. One should choose a superconductor with a small value of $\langle \omega \rangle$, that is, with a low phonon mode. The perfectly realistic situation is that of $\Gamma \gg \langle \omega \rangle$ in the region of small d. The critical temperature is described by Eq. (31) with $\langle u \rangle \approx \langle \omega \rangle$. Let the thickness "d" increase gradually and consider the dependence $T_c(d)$. Gradual increase of d means the gradual decrease of the coupling. An increase in "d" results in a decrease in Γ. The critical temperature does not change noticeably until the inequality $\Gamma \gg \langle \omega \rangle$ holds. Then we come to the region $d > d_c$ the quantity $\langle u \rangle \approx \Gamma$ (see Fig. 2), and the subsequent increase of d and the corresponding decrease of Γ result, according to Eq. (31), in the essential dependence $T_c(d)_{|d>d_c}$. According to Eq. (31), T_c

increases with an increase of d in the region $d > d_c$. The increase of $T_c(d)$ described by Eq. (31) continues until $\Gamma \gg T_c$. When $\Gamma \lesssim T_c$, Eq. (31) is not applicable but then we can use Eqs. (40) and (41). It is easy to see that if $\Gamma \ll T_c$, the critical temperature tends to T_c' (if it is possible to neglect the term $g_\alpha^{(1)}$ which is due to the oxide, then $T_c' = T_c^\alpha$). Hence, an increase in d gradually leads to the value $T_c = T_c'$, and this equality is natural for large value d.

Hence the change of T_c is described by the following regularity. If $d < d_c$, the critical temperature remains almost constant. An increase in d in the region $d > d_c$ is accompanied by a noticeable increase of T_c up to T_c'. We have seen that $\Gamma(d_c) \approx \langle\omega\rangle$. If function $g(\omega)$ and the quantities ν^α and ν^β are known, it is possible to calculate $\langle\omega\rangle$ and $\Gamma(d_c)$. Then the dependence $T_c(L_\beta)$ (if $d = d_c$) can be verified experimentally.

3. Maximum T_c

We see from Eqs. (26) and (31) that the proximity effect results in a decrease of T_c. The dependence $T_c(L_\beta)$, which is caused by the proximity effect, can be non-monotonic (see below), but the inequality $T_c < T_c'$ always holds. Hence, the contributions of the terms I_1 (if $g_\alpha^{(1)} > 0$) and I_2 to a change of T_c (see Eq. (14)) have opposite signs. If L_β is small ($L_\beta \sim a$, a is atomic distance) the contribution of I_2 is small, because $(\Gamma_{\alpha\beta}/\Gamma) \sim L_\beta/L_\alpha$. The term I_1 also contains the small parameter λ_1/λ_α (see Eq. (24)), but a perfectly realistic situation is when the contribution of I_1 dominates. Then the covering results in an increase of T_c relative to T_c^{α} (in the region of $L_{\beta} \sim a$). If $L_{\beta} >> a$, the term I_2 becomes more important, because the effect of the change of the phonon spectrum on the electron system the α film is limited by the atomic distance. On the other hand, the term I_2 increases as L_{β} increases. Hence the quantity $\partial T_c/\partial L_{\beta}|_{L_{\beta}>>a}$ depends mainly on I_2 and T_c decreases with increase of L_{β}. Therefore we come to the conclusion that T_c should have a maximum $T_{cm} \sim T_c'$ in the region $L_{\beta} \sim a$.

A maximum of T_c in the region of small L_{β} was observed experimentally in[1-4] (see Fig. 3). Mikheeva et al.[2] have developed a very precise method of determination of the thickness of films. They observe $T_{c_{max}}$ in the Mo-C, Te-C and V-C systems, when $L_C \approx 5$ Å. The system Al-SiO was considered by Sixl.[3] The dependence T_c on the thickness of the SiO coating is characterized by $T_{c_{max}}$, when $L_{SiO} \approx 3$ Å. The same result was obtained by Orlov et al. by the investigation of Pb-Ge and Pb-Si systems.[4] The first observation of $T_{c_{max}}$ was made by Naugle[1] for the system Tl-Ge in the region of small $L_{Ge} \approx 10$ Å. Note that the investigations[2-4] are characterized by a more precise determination of the thickness of the films. It is worth noting that the existence of this maximum T_c cannot be explained by the proximity effect, because the proximity effect in the absence of a change of the phonon spectrum results in the inequality: $T_c < T_c^{\alpha}$, which contradicts the experimental data.[1-4]

The appearance of $T_{c_{max}}$ can be explained by the combination of the change of the phonon spectrum (term I_1) and the proximity effect (term I_2). Note that the absence of $T_{c_{max}}$ in the Sn-Ge[1] Pb-C[4] systems is connected with the small value of the function $g_\beta^{(1)}$.

According to our explanation, an increase of L_α does not affect noticeably the position of the maximum, but it does affect the value of $T_{c_{max}}$. This conclusion is consistent with the experimental data.[2-4]

If β film is not size-quantizing, the critical temperature is described by Eq. (31), or Eq. (38) (if $\Gamma \ll \langle\omega\rangle$; $\Gamma \ll T_c$). In this case ρ is described by Eq. (34) and depends very significantly on L_β. The quantity Γ depends on L_β also (see Eq. (6'), (32)), but the dependence $\rho(L_\beta)$ is more essential. The dependence ρ on L_β leads to monotonic decrease of T_c with an increasing in L_β. If β is size-quantizing film, the situation changes drastically (see Sec. V).

V. SIZE QUANTIZATION

1. Density of States

According to Eqs. (31), (32), T_c depends on the function $\Gamma_{\alpha\beta}$, which (see Eq. (6)), is coupled directly with the density of the states of the non-metallic film.

Consider now a realistic situation when semimetal (or semiconductor) film is size-quantizing. As is well-known (see, e.g., review[10]) the size quantization (SQ) is very sensitive to the structure of the film and to the quality of its surface. The best conditions for SQ are realized in semimetal film, where a number of factors (such as low electron density, and a small value of the transverse effective mass) cause the de Broglie wavelength to exceed greatly the atomic distance and it is this which makes the surface, in fact specular.

SQ is observed experimentally by the investigation of the films of Bi[6,7], Sb[8], InSb[9]. The observation of this effect in thin metallic films is also possible (see, e.g., Refs. [25,26]), but is considerably more complicated.

Consider a thin semimetal (or semiconductor) film. SQ results in the situation, when the energy $\varepsilon(\vec{\kappa},n)$ is determined by the longitudinal two-dimensional quasimomentum $\vec{\kappa}$ and by the transverse quantum number n. Instead of a Fermi surface we have a group of two-dimensional subbands.

The density of states can be written in the form (see e.g., Ref. [10]).

$$\nu_\beta = \sum_i \nu_\beta^i$$

where

$$\nu^i_\beta = \frac{1}{L_\beta}\frac{\tilde{m}^i}{\pi\hbar^2} r \quad ; \quad \tilde{m}^i = (m^i_1, m^i_2)^{1/2} \tag{42}$$

The summation is taken over the filled subbands; r is the number of two-dimensional valleys. The dispersion relation has in the effective mass approximation the following form:

$$\varepsilon(\vec{\kappa}, n_1) = \frac{\hbar^2}{2m_1}(\kappa_x - \kappa_{xo})^2 + \frac{\hbar^2}{2m_2}(\kappa_y - \kappa_{yo})^2 + \varepsilon_1 \tag{42'}$$

If only one subband is filled, the density of states is equal to:

$$\nu_\beta = \frac{1}{L_\beta}\frac{\tilde{m}}{\pi\hbar^2} r \tag{43}$$

Indeed if only one subband is filled, the number of states in the region κ, $\kappa+d\kappa$ is equal to $4\pi\kappa d\kappa S/(2\pi\hbar)^2$ (S is the area of the film; $\varepsilon = \kappa^2/2m$). The density of states (per unit volume) is equal to $\nu = mL^{-1}/\pi\hbar^2$ (L is the thickness of the film) and decreases with increase of L. If the film is characterized by several filled subbands with the dispersion relation Eq. (42') and by several valleys, we come to Eq. (42).

The number of filled subbands ν depends on the electron concentration "n". If $n < L^{-3}$, only the lowest subband is filled (more exact conditions are given below.) For example, in Bi films only one subband remains filled up to $L \sim 5 \cdot 10^2$ Å. The film, which remains a three-dimensional system in the coordinate space ($L \gg a$) becomes a two-dimensional system in the momentum space, and this results in a peculiar behavior.

An increase of L leads to a decrease of the distance between transverse levels. When L becomes equal to some value L_c, the next subband begins to be filled, and this is accompanied by a jump of the density of states. Hence, the density of states is an oscillating function of L.[10,27,28]

The oscillations of the density of states ν_β result, according to Eqs. (6), (31), (32), and (42) in a non-monotonic dependence $T_c(L_\beta)$. The values $L_{\beta c}$ correspond to jumps of the density of states. They depend on the dispersion relation. To estimate their values we use the simple model, when the longitudinal motion is described by the relation

$$\varepsilon = \kappa^2/2\tilde{m} \quad ; \quad \Delta\varepsilon_\perp^{(1)} = 3\pi^2\hbar^2/2m_\perp L_\beta^2 \tag{44}$$

(the potential box or the quantization $\kappa_\perp = \pi\hbar n/L_\beta$). The second subband begins to be filled if L_β satisfies the condition

$$\Delta\varepsilon^{(1)} = \varepsilon_F \tag{44'}$$

where ε_F is the Fermi level.

It is necessary to take into account the fact that $\kappa_F = (2\tilde{m}\ \varepsilon_F)^{1/2}$ also depends on L_β. The electron concentration n is equal to $n = \kappa_F^2/2\pi\hbar^2 L_\beta$. Therefore:

$$\kappa_F^2 = 2\pi\hbar^2 nL_\beta \tag{44''}$$

Based on Eqs. (44) - (44"), we obtain the following value for $L_{\beta c}{}^{(1)}$:

$$L_{\beta c}^{(1)} = a(\widetilde{m}/m_{\perp} n)^{1/3} \quad ; \quad a \approx 1.7 \tag{45}$$

If the condition $nL_{\beta}{}^3 < (\widetilde{m}/m_{\perp})$ (see Eq. (45)) is satisfied, only the lowest subband is filled. Let us estimate the value $L_{\beta c}{}^{(2)}$, which corresponds to the next rapid change of T_c. Generally speaking, values of the masses for different subbands can be different. Solving the equation

$$\begin{aligned} \Delta E^{(2)} &= \kappa_{F2}^2/2\widetilde{m}_2 \\ (\kappa_{F2}^2/2\widetilde{m}_2) + \Delta E^{(1)} &= \kappa_{F1}^2/2\widetilde{m}_1 \\ \kappa_{F1}^2 + \kappa_{F2}^2 &= 2\pi\hbar^2 nL \end{aligned} \tag{46}$$

results in the following value for $\Delta L = L_{\beta c}{}^{(2)} - L_{\beta c}{}^{(1)}$:

$$\Delta L = a\left(\frac{\widetilde{m}_1}{m_{\perp} n}\right)^{1/3} [b(1 + c(\widetilde{m}_2/\widetilde{m}_1))^{1/3} - 1] \tag{47}$$

where a, b, c $\sim$ 1 (in the model of the box $a \approx 1.7$, $b \approx 1.4$, $c \approx 0.6$). For example, $\Delta L \approx 30$ Å for Sb films.[8,10] The quantity ΔL in the Bi films can be changed in the region 10^2 Å $< \Delta L < 4 \cdot 10^2$ Å depending on the structure of the film.[6] We see that $\Delta L \neq L_1$ and the oscillations are aperiodical.

2. Non-Monotonic Behavior of T_c

The critical temperature is described by Eqs. (26), (31), and (40). If the "β" film is a semimetal (or semiconductor) film with SQ, the densities of states ν_α and ν_β are described by Eq. (33) and Eq. (42).

SQ results in the situation when ρ and, consequently T_c change step-wise. If L_β becomes equal to $L_{\beta c}{}^{(1)}$ (see Eq. (45)), ν_β increases very rapidly and this leads to a rapid decrease of T_c. The detailed picture of the dependence $T_c(L_\beta)$ varies with the conditions of the experiment. Consider the case when the ratio L_β/L_α = const. If $L_\beta < L_{\beta c}{}^{(1)}$, the quantity ν_β decreases with the increase of L_β (see Eq. (42)), and one can observe an increase in T_c (of course, it always holds that $T_c < T_c'$). When L_β becomes equal to $L_{\beta c}{}^{(1)}$ ν_β makes a step-wise increase, and T_c decreases, according to Eq. (31) (if $\Gamma \gg T_c$). Hence an appearance of a maximum of T_c is possible. This second maximum differs from the maximum of T_c in the region $L_\beta \sim a$ (see Sec. III) by its nature. The subsequent increase of L_β in the region $L_\beta > L_{\beta c}{}^{(1)}$ again leads to a decrease of ν_β and an increase of T_c, and this continues until $L_\beta = L_{\beta c}{}^{(2)}$, and so on (see Fig. 4). Hence SQ leads to the possibility of oscillations of T_c.

If L_α = const (this case corresponds to the typical experimental situation; see Refs. [1-4]), then (see Eqs. (32'), (42))

$$\rho \simeq \frac{\pi \tilde{m}_\beta r \nu_o}{m_\alpha p_F^\alpha L_\alpha} \tag{48}$$

(cf. (Eq. (34)), where ν_o is the number of filled subbands

We would like to emphasize, that SQ leads to peculiar dependence $\rho(L_\beta)$ (cf. Eq. (34)). According to Eq. (48), $\rho(L_\beta)$ is a step-wise function, and, if $L_\beta < L_{\beta c}^{(1)}$ or $L_{\beta c}^{(1)} < L_\beta < L_{\beta c}^{(2)}$ etc. (see Eq. (45), (47)), the quantity ρ does not depend on L_β at all. Then the dependence $T_c(L_\beta)$ is caused only by the dependence $\Gamma(L_\beta)$ (see Eqs. (6'), (38), it is assumed, that $\Gamma \ll \langle\omega\rangle$). According to Eqs. (6'), (32), (38), the increase of L_β results in increase T_c. This increase continues until $L_\beta = L_{\beta c}^{(1)}$. Then we get $\nu_o = 2$, and T_c makes a step-wise decrease and so on. Hence the dependence $T_c(L_\beta)$ becomes non-monotonic.

Moreover, if $\Gamma \gg \langle\omega\rangle$, one should use Eq. (39). Then an increase of L_β does not affect T_c until the decrease of $\Gamma_{\beta\alpha}$ (see Eq. (6')) gradually results in situation, when $\Gamma < \langle\omega\rangle$. Then the dependence $T_c(L_\beta)$ is described by Eqs. (31), (38), and (40), and $T_c(L_\beta)$ becomes the increasing function. Therefore, SQ leads to a nonmonotonic dependence $T_c(L_\beta)$. Note, that the distance between the maximum T_c in the region $L_\beta \sim a$ (see Sec. III) and the maximum T_c, if $L_\beta = L_{\beta c}^{(1)}$, is not equal to the period of oscillation, because they have different origins. As is known (see e.g., Ref. [10]) the thin film can be size-quantizing if its thickness $L_\beta > 10$ Å, and, hence, the described increase of T_c can be observed, if $L_\beta > 10$ Å. Therefore, T_c has a minimum in the region $L_\beta \sim 10$ Å.

The non-monotonic dependence $T_c(L_\beta)$ (see Sec. III, Eqs. (6'), (38), and (48)) has been observed experimentally (see Fig. 1) by the investigation of the Mo, Te, V films covered by C[2] and the systems Al-SiO[3] and Pb-Si.[4] (For a general description of the experimental situation see Ref. [32]. The oscillations in the region $L_\beta > 10$ Å are observed only in those cases, when the β films are semiconductor or semimetal). Note that SQ is not a universal phenomenon and it is very sensitive to the structure of the films (see, e.g., Ref. [10]). It is not surprising that the increase of $T_c(L_\beta)$ has not been discovered by the investigation of some systems (e.g., Pb-Ge, Pb-C,[4] Tl-Ge and Sn-Ge[1]). In the absence of SQ, T_c decreases with increased L_β, in accordance with Eqs. (34) and (38). Of course, it is necessary to have the information about phonon and electron spectra of the films. Then it is possible to make the detailed comparison with experimental data.[1-4] We present the theoretical dependence $T_c(L_\beta)$ (see Fig. 5) for some values of the parameters. The quantity $\Gamma^{\beta\alpha}$ is described[11] by Eq. (6') (see also Ref. [22]). The S-N sandwiches with Cu and Ag normal layers have been studied in Ref. [12]. According to measurements in,[12] the best description of the experimental data is obtained, if the values $\Gamma_M{}^{\beta\alpha}$ are: $\Gamma_M{}^{\beta\alpha} = 8$ mv ($L_\beta = 100$ Å) $\Gamma_M{}^{\beta\alpha} = 4$ mv ($L_\beta = 200$ Å), $\Gamma_M{}^{\beta\alpha} = 2$mv ($L_\beta = 400$ Å, cf Eq. (6')), where $\Gamma_M{}^{\beta\alpha} \equiv \Gamma^{\beta\alpha}$ for metallic film. We discussed the method of obtaining the value $\Gamma^{\beta\alpha}$ (see IV.2), which can be applied, if β film is SM or SC film also. It is possible to estimate the values $\Gamma_{SQ}{}^{\beta\alpha}$ ($\Gamma_{SQ}{}^{\beta\alpha} \equiv \Gamma^{\beta\alpha}$ for size-quantizing film), using Eq. (6') and data:[12]

$$\Gamma_{SQ}^{\beta\alpha} \sim p_{F_\perp}\sigma/m_\perp L_\beta \sim \pi\hbar\sigma/m_\perp L_\beta^2 \sim \Gamma_M^{\beta\alpha}(a/L_\beta)(m_\perp^*/m_\perp)$$

Here $m_\perp^*$ and $m_\perp$ are the effective transverse masses of the metallic and SM (or SC) films. We consider the case, then only the lowest subband is filled and $p_{F_\perp} \sim \pi\hbar/L_\beta$; in the metal film $p_F \sim \hbar/a$. The smallness a/L_β can be compensated by the ratio $m_\perp^*/m_\perp$. For example, in Bi film $m_\perp = 0.01\ m_e$. If, for instance $L_\beta = 75$ Å, the value $\Gamma^{\beta\alpha}$ in Bi film is $\Gamma^{\beta\alpha} \sim 15$ mv. We present the theoretical dependence $T_c(L_\beta)$ (see Fig. 5) for some values of the parameters.

3. Charge Density Waves

SQ results in appearance of the peculiar charge density waves. This problem has been considered by Kokotov and author in Ref. [29]. This kind of instability can affect the properties of S-SM and S-SC systems.

Consider the case, when only the lowest subband is filled. The film is characterized by the Fermi line (see above, Sec. V.1). If several subbands are filled, there is a set of Fermi lines. A perfectly realistic situation is one in which the Fermi line has linear sections. For example, the curvature radius of the definite sections of the Fermi line in Bi films (see [6]) is larger than the dimension of the Fermi line itself and even the dimension of the Brillouin Zone. These sections can be regarded as straight lines with high degree of accuracy.

The electron-phonon interaction leads to the instability of the lattice in the presence of the linear sections of the Fermi line.[29] The calculation of the polarization operator

$$\Pi(q) = 2i \int dpG(p+q/2)\, G(p-q/2)$$

$$(q = (\vec{q}_{\parallel}, \omega),\ p = (\vec{\kappa}, \varepsilon)$$

leads to the expression[29]

$$\Pi \sim q_o \ln (8\, \varepsilon_F / i\omega) \tag{49}$$

which contains the logarithmic singularity (it is assumed that the linear section corresponds to the region $|\kappa_x| < q_o$). The presence of a logarithmic singularity in Π leads to the appearance of an imaginary pole in the phonon Green function $D = D_o^{-1} - g\, \Pi\, g_e$ ($g^{\varepsilon} = (1 + V\Pi)^{-1}$) (see e.g., Ref. [30]) and to lattice instability. The temperature T_p, which corresponds to the appearance of static-deformation waves and the structural transition is

$$T_p \approx \varepsilon_F\, e^{-1/\lambda} \tag{50}$$

The considered SM and SC films are characterized by a small value of ε_F (e.g., for Bi films $\varepsilon_F \approx 10^{-2}$ eV) relative to ε_F of metals. That is why the value T_p is small ($T_p \lesssim 1°$). The minimum of the resistance, which was observed in an experimental investigation of thin Bi films in the low temperature region,[31] is described by Eq. (50).

The temperature $T = T_p$ is characterized by the appearance of a static-deformation wave. If $T > T_p$, the gradual decrease of T leads to the softening of the phonon mode. Because of smallness of T_p in the SM and SC size-quantizing films, one can consider the situation when T_c of the S-SM or S-SC system is larger than T_p. Then the region $T \sim T_c$ is characterized by the existence of low phonon mode, and this results in an increase of the constant describing the electron-phonon interaction in the "β" film. The value of this constant can exceed the value of the Coulomb pseudopotential and the effective constant λ_β becomes different from zero.

VI. THE CASE $\lambda_\beta \neq 0$

Previously we considered the case $g_\beta = 0$. Suppose that film "β" is characterized by a non-zero value of g_β. More exactly, it means that the value of the effective constant λ_β describing the electron-phonon interaction in film "β" exceeds the value of the Coulomb pseudo-potential μ^*. Then one should write the following equations $T = T_c$; cf. Eqs. (7) and (8)

$$C^\alpha(\omega_n) = C^\alpha_{ph}(\omega_n) + \Gamma_{\alpha\beta}\, C^\beta(\omega_n)/(|\omega_n| Z^\beta) \tag{51}$$

$$C^\beta(\omega_n) = C^\beta_{ph}(\omega_n) + \Gamma_{\beta\alpha}\, C^\alpha(\omega_n)/(|\omega_n| Z^\alpha) \tag{52}$$

where the additional term is equal to:

$$C^\beta_{ph}(\omega_n) = \pi T \sum_{\omega_{n'}>0} \int d\omega\, g_\beta(\omega) \frac{\omega^2}{\omega^2+(\omega_n-\omega_{n'})^2} \cdot \frac{C^\beta(\omega_{n'})}{|\omega_{n'}| Z^\beta(\omega_{n'})} \tag{53}$$

Equation (52) can be written in the form:

$$C^\beta(\omega_n) = \Gamma_{\beta\alpha}\, C^\alpha/(|\omega_n| Z^\alpha) + R \tag{54}$$

where

$$R = \Gamma_{\beta\alpha}\, \pi T \sum_{\omega'_n} \int d\omega\, g_\beta(\omega) \frac{\omega^2}{\omega^2+\omega^2_{n'}} \cdot \frac{C^\alpha(\omega_{n'})}{\omega^2_{n'}\, Z^\alpha(\omega_{n'})\, Z^\beta(\omega_{n'})} \tag{55}$$

Therefore, we see that Eq. (54) for the function C^{β} contains, besides the usual term (see Eq. (8)), the additional term R.

Substituting then Eqs. (54) and (55) into Eq. (51) and performing calculations by analogy with the derivation of Eq. (13), we arrive at the following equation:

$$1 = 2\pi T \sum_{\omega_n > 0} \int d\omega\, g_{\alpha}(\omega) \frac{\omega^2}{\omega^2 + \omega_n^2} \cdot \frac{K(\omega_n)}{\omega_n Z^{\alpha}(\omega_n)} \tag{56}$$

where

$$K(\omega_n) = \frac{1}{S(\omega_n)} + \frac{\Gamma_{\alpha\beta}\Gamma_{\beta\alpha}}{\omega_n Z^{\beta}(\omega_n) S(\omega_n)} \times 2\pi T \sum_{\omega_{n'} > 0} \int d\omega\, g_{\beta}(\omega) \frac{\omega^2}{\omega^2 + \omega_{n'}^2} \cdot \frac{1}{\omega_{n'}^2 Z^{\alpha}(\omega_{n'}) Z^{\beta}(\omega_{n'}) S(\omega_{n'})} \tag{57}$$

Here

$$S(\omega_n) = 1 - \Gamma_{\alpha\beta}\Gamma_{\beta\alpha} / \omega_n^2 Z^{\alpha}(\omega_n) Z^{\beta}(\omega_n) \tag{58}$$

According to Eqs. (4) and (5), we obtain

$$S^{-1}(\omega_n) = Z^{\alpha}(\omega_n) Z^{\beta}(\omega_n) / (1 + \Gamma/\omega_n) \tag{59}$$

According to Eqs. (13), (15), (57) and (59), Eq. (56) can be reduced to the form:

$$1 = \int d\omega\, g_\alpha(\omega) \ln \frac{2\omega\gamma}{\pi T_c} - \frac{\Gamma_{\alpha\beta}}{\Gamma} L_\alpha + \frac{\Gamma_{\alpha\beta}\Gamma_{\beta\alpha}}{\Gamma^2} L_\alpha L_\beta \tag{60}$$

where

$$L_i = 2\pi\, \Gamma\, T \sum_{\omega_n > 0} \int d\omega\, g_i(\omega) \frac{\omega^2}{\omega^2+\omega_n^2} \cdot \frac{1}{\omega_n(\omega_n+\Gamma)} \tag{61}$$

$$i \equiv \{\alpha, \beta\}$$

Using Eq. (19) and summing in Eq. (61) over ω_n, we obtain

$$\ln \frac{T_c}{T_c'} = -\frac{\Gamma_{\alpha\beta}}{\lambda_\alpha \Gamma} \int d\omega\, g_\alpha(\omega)\, f(\omega,\Gamma,T_c) + \frac{\Gamma_{\alpha\beta}\Gamma_{\beta\alpha}}{\lambda_\alpha \Gamma^2} \int d\omega\, g_\alpha(\omega)\, f(\omega,\Gamma,T_c) \cdot \int d\omega\, g_\beta(\omega)\, f(\omega,\Gamma,T_c) \tag{62}$$

where (cf. Eqs. (26) and (27))

$$f(\omega,\Gamma,T_c) = \frac{\omega^2}{\omega^2+\Gamma^2} \ln \frac{2\Gamma\gamma}{\pi T_c} + \frac{\Gamma^2}{\omega^2+\Gamma^2} \ln \frac{2\omega\gamma}{\pi T_c} \tag{63}$$

and T_c' is defined by Eq. (19); it is assumed that $\Gamma \gg T_c$.

Equation (62) can be written in the form:

$$\ln \frac{T_c}{T_c'} = -\frac{\Gamma_{\alpha\beta}}{\Gamma} \ln \frac{2\gamma\langle u\rangle_\alpha}{\pi T_c} + \phi \tag{64}$$

where

$$\phi = \frac{\Gamma_{\alpha\beta}\Gamma_{\beta\alpha}}{\Gamma^2} \lambda_\beta \ln \frac{2\gamma\langle u\rangle_\alpha}{\pi T_c} \ln \frac{2\gamma\langle u\rangle_\beta}{\pi T_c} \tag{65}$$

The quantity u is defined by Eqs. (30), (35), and $\langle u\rangle_i$ denotes the mean value in the sense

$$\ln \frac{\langle u\rangle_i}{T_c} = \frac{1}{\lambda_i} \int d\omega\, g_i(\omega) \ln \frac{u}{T_c}$$

If $\Gamma \ll \langle\omega\rangle_\alpha$, $\Gamma \ll \langle\omega\rangle_\beta$ then $\langle u\rangle_i = \Gamma$ (see above). Then

$$\phi = \lambda_\beta \frac{\Gamma_{\alpha\beta}\,\Gamma_{\beta\alpha}}{\Gamma^2}\left(\ln \frac{2\Gamma\gamma}{\pi T_c}\right)^2 \tag{66}$$

The critical temperature can be evaluated from Eq. (64). Note that the terms on the right-hand side of Eq. (64) have opposite signs. Therefore the inequality $\lambda_\beta \neq 0$ results in an increase of T_c relative to the situation, when $\lambda_\beta = 0$.

If, for example, λ_β is small $(T_c^\beta \ll T_c^\alpha;\ \lambda_\beta \approx \ln^{-1}(\langle\omega\rangle_\beta/T_c^\beta))$ and $\Gamma_{\alpha\beta} \ll \Gamma$, the increase of T_c due to the second term on the right-hand side of Eq. (64) is equal to:

$$\frac{\Delta T_c}{T_c} \sim \lambda_\beta \frac{\Gamma_{\alpha\beta}}{\Gamma}\left(\ln \frac{2\Gamma\gamma}{\pi T_c}\right)^2 \tag{67}$$

The smallness of the factor $\lambda_\beta \Gamma_{\alpha\beta}/\Gamma$ is compensated to some extent by the large logarithmic factor $(\ln 2\Gamma\gamma/\pi T)^2$. If, for example, $\langle\omega\rangle_\beta \sim 10$ mV, $T_\alpha{}^c \sim 10^{-3°}$, $\Gamma_{\alpha\beta}/\Gamma \sim 0.1$, $\Gamma/T_c \sim 10$, we obtain $(\Delta T_c/T_c) \sim 5\%$. Hence, we see that the inequality $\lambda_\beta \neq 0$ leads to an increase of T_c.

VII. DISCUSSION

The considered S-SM and S-SC systems are characterized by a number of parameters. A variety of the parameters allows one to change the properties and, first of all, T_c in the desired direction.

The critical temperature of S-SM and S-SC systems in the presence of size quantization is described by Eqs. (19), (31), (41) and (48). In the absence of SQ one should use Eq. (34) instead of Eq. (48). The relation $T_c' > T_c^\alpha$ is caused by the change of the phonon spectrum (see Sec. III). On the other hand, the proximity effect tends to decrease the critical temperature. SQ leads to oscillations of the density of states and, hence, to oscillations of T_c. The combination of the mentioned factors results in complex non-monotonic dependence $T_c(L_\beta)$. The estimates (see above) show that the quantitative change of T_c can be very noticeable. Equations (19), (29)-(32'), (48) allow to carry out the detailed calculations. These equations express the value of T_c in terms of measured quantities. Namely, the value of T_c depends on the function $g(\omega)$ and the properties of the dispersion relation of film in the presence of SQ.

As is well-known, there are several methods, allowing to determine the function $g(\omega)$. The most powerful method is the method of tunnel spectroscopy. A very interesting investigation was carried out by Chaikin, Arnold and Hansma.[12] The authors of Ref. [12] have studied the system superconductor-normal metal. They took advantage of the proximity effect to get information about the quantity λ_n describing the electron-phonon interaction (EPI) and the phonon spectrum of normal metal.

A similar investigation of semimetal and semiconductor films would be very interesting. One can get the same interesting information about the EPI and the phonon spectrum in these types of solids. Moreover, the change of the thickness of the oxide (see, e.g., Ref. [33]) allows one to get information about the function $g_\alpha^{(1)}$ (see Sec. III).

Investigation of the dispersion relation in size-quantizing thin films is a very interesting problem. This problem was discussed by B. Kokotov and the author in Ref. [29]. As was mentioned above, in the presence of SQ, electrons are characterized not by the Fermi surface, but by the Fermi line. The Fermi line can differ from the usual section through a three-dimensional Fermi surface because of the specifics of the film state brought about by sputtering conditions and, moreover, because, strictly speaking, the transverse quasimomentum is not defined for these thin films. One can suggest (see Ref. [29]) several methods (e.g., investigation of sound adsorption in a magnetic field, absorption of an electromagnetic field, and so on) which allow one reconstruct the Fermi line by using experimental data. The development of this direction is important, because it allows one to study the properties of the film state. In our case, it will be possible to calculate the density of states ν_β and the corresponding contribution to the change T_c.

The best conditions for observations of SQ are realized in the SM and SC films. In principle, it is possible, although more complicated, to observe SQ in thin metallic films.[25,26] The effect of SQ on T_c was considered by Blatt[27] and by Tavger and the author.[28]

It is supposed in this paper that the metallic film is not size-quantizing. SQ of the metalic film with a number of filled subbands can also affect T_c of S-SC and S-SM systems. This additional mechanism was considered in Ref. [38]. It was assumed that it is possible to separate the variables, and this corresponds to the specular reflection. The change of T_c is caused by the boundary conditions.[38] This mechanism is not connected with oscillations of the density of states of SM or SC films and with EFI and is very sensitive to the quality of the metallic film.

We have discussed (see above) the effect of different factors on the dependence $T_c(L_\beta)$. The non-monotonic dependence $T_c(L_\beta)$ was observed experimentally. A whole set of experimental data can be explained in terms of the present theory (see Sec. IV.3, and VI).

Hence, we see that the investigation of S-SM and S-SC systems in the presence of SQ leads to the possibility of the peculiar change of T_c. The development of this direction and subsequent experiments promise to be very interesting.

<u>Summary</u>

1. We have considered S-SM and S-SC systems containing thin superconducting and semimetal (or semiconductor) films. Using the method of the thermodynamic Green function, we considered the influence of the proximity effect, the size quantization and the change of the phonon spectrum on T_c.

2. The dependence of T_c on the thickness of non-metallic films is non-monotonic. One can observe $T_{c_{max}}$ in the region $L_\beta \sim a$.

3. Equation (31) describing T_c in the framework of the phonon model in the presence of the proximity effect has been obtained. The peculiar dependence T_c (d_{bar}) was discussed.

4. The size quantization leads to oscillations of $T_c(L_\beta)$.

5. The features of the dispersion relation in a size-quantizing film, the appearance of the charge density waves were discussed.

6. The increase of T_c caused by the inequality $\lambda_\beta \neq 0$ was considered.

7. The experimental data are discussed. It will be interesting to carry out new experiments (see Sec. VI).

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FIGURE CAPTIONS

Fig. 1. The temperature self-energy parts.

Fig. 2. Universal function $y(t)$, $y = u/\Gamma$, $t = \omega/\Gamma$.

Fig. 3. Functional dependence $T_c(L_\beta)$ for a) 1 - Al-SiO[3], $L_\alpha = 21Å$; 2 - Mo-C[2], $L_\alpha = 60Å$; 3 - Te-diamond[2], $L_\alpha = 50Å$, b) Al-Si[3], $L_\alpha = 70Å$, c) 1 - Pb-Ge[4], 2 - Pb-Si[4], $\delta T_c = T_c - T_c^\alpha$.

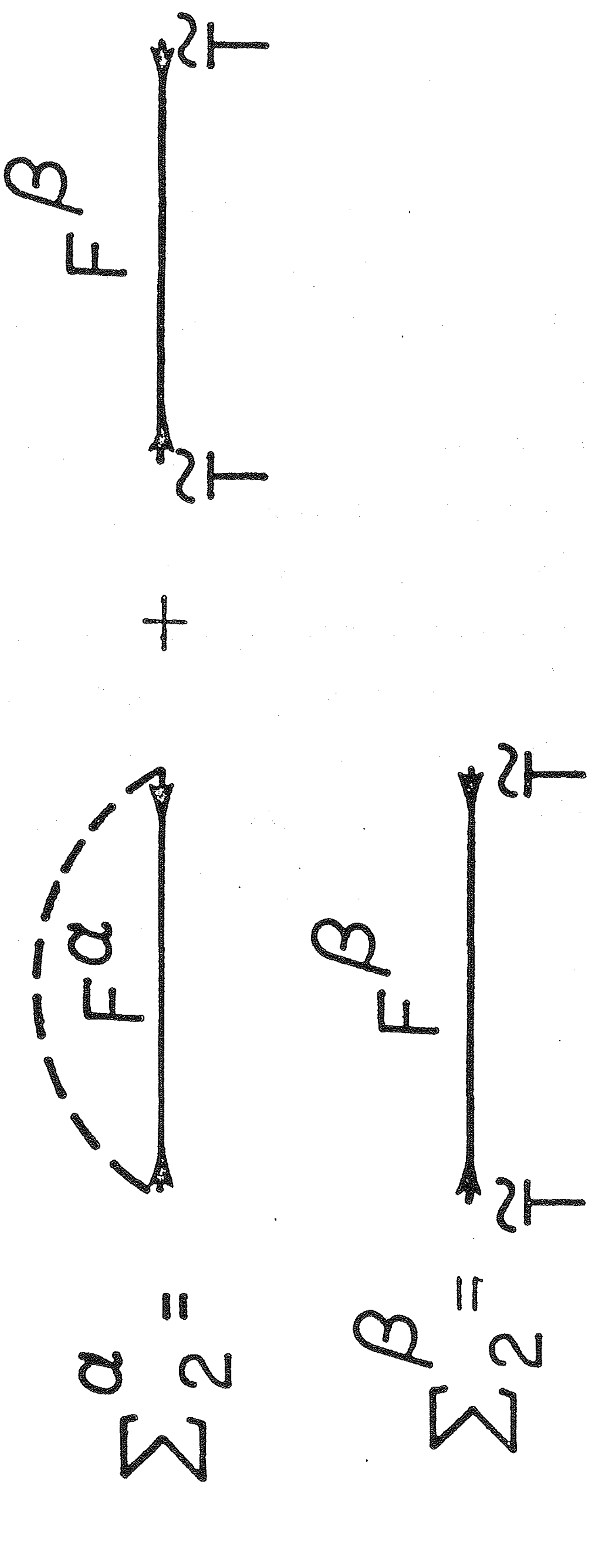
Fig.4. The dependence $T_c(L_\beta)(L_\alpha/L_\beta = \text{const}$, $\tau_c = T_c/T_c^\alpha$, $\tilde{\Gamma} = \Gamma/T_c^\alpha$, $\tau_c' = T_c'/T_c^\alpha)$ for parameters: $\rho(L_\beta = 20Å) = 0.05$, $\Gamma(L_\beta = 20Å) = 40$, $\lambda'/\lambda = 0.02$, $\langle\omega\rangle / T_c^\alpha = 10^2$.

Fig. 5. Transition temperature T_c versus L_β (L_α = constant) for:

a) $\rho = 0.05$, $\tilde{\Gamma}(L_\beta = 30Å) = 10$, $(\lambda_1/\lambda) = 0.09$, $\langle\omega\rangle/T_c^\alpha = 15$;

b) $\rho = 0.05$, $\tilde{\Gamma}(L_\beta = 20Å) = 20$, $\lambda_1/\lambda = 0.01$, $\langle\omega\rangle/T_c^\alpha = 150$;

c) $\rho = 0.1$, $\tilde{\Gamma}(L_\beta = 20Å) = 10$, $\lambda_1/\lambda = 0.04$, $\langle\omega\rangle/T_c^\alpha = 10^2$.



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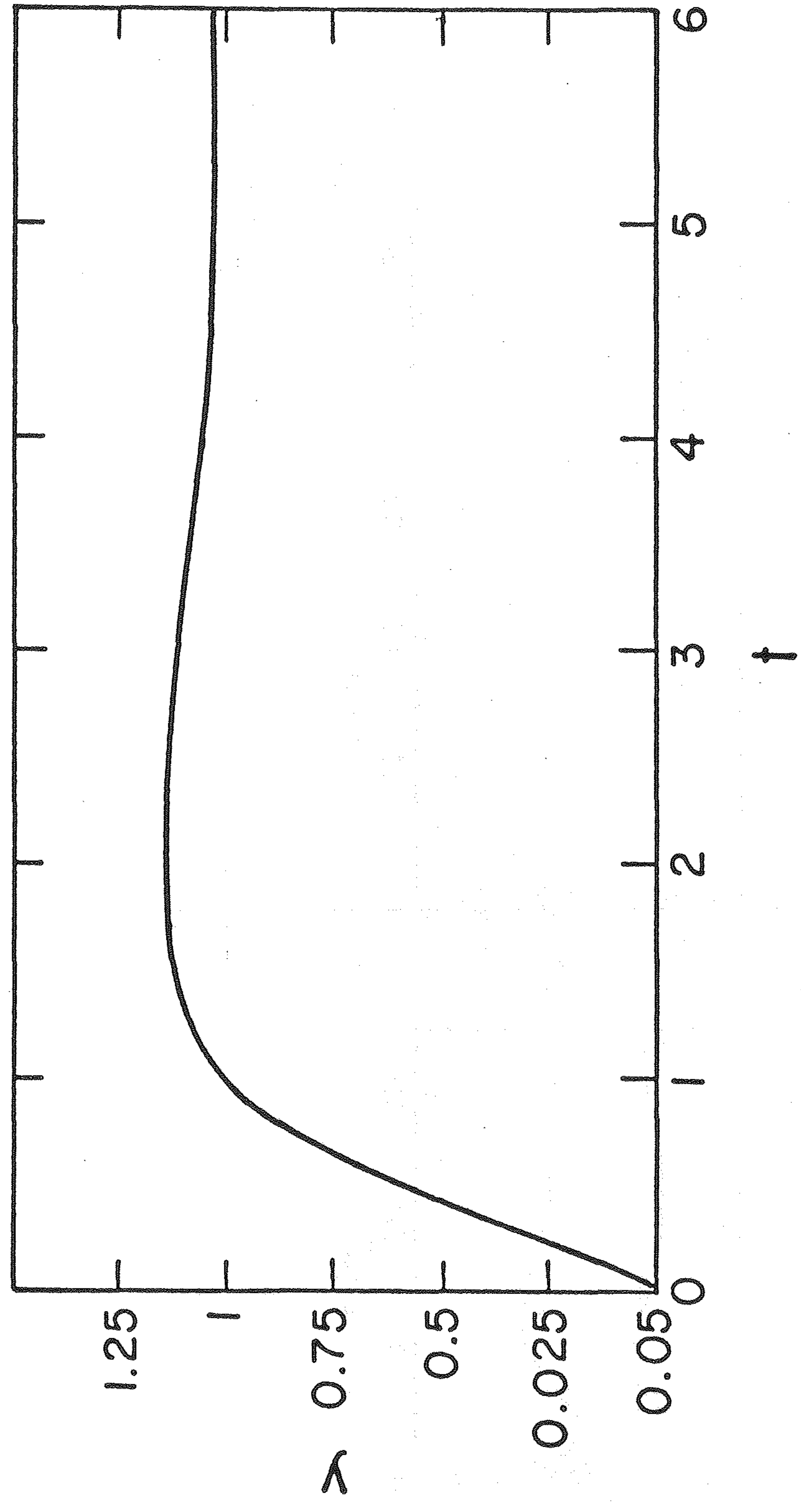

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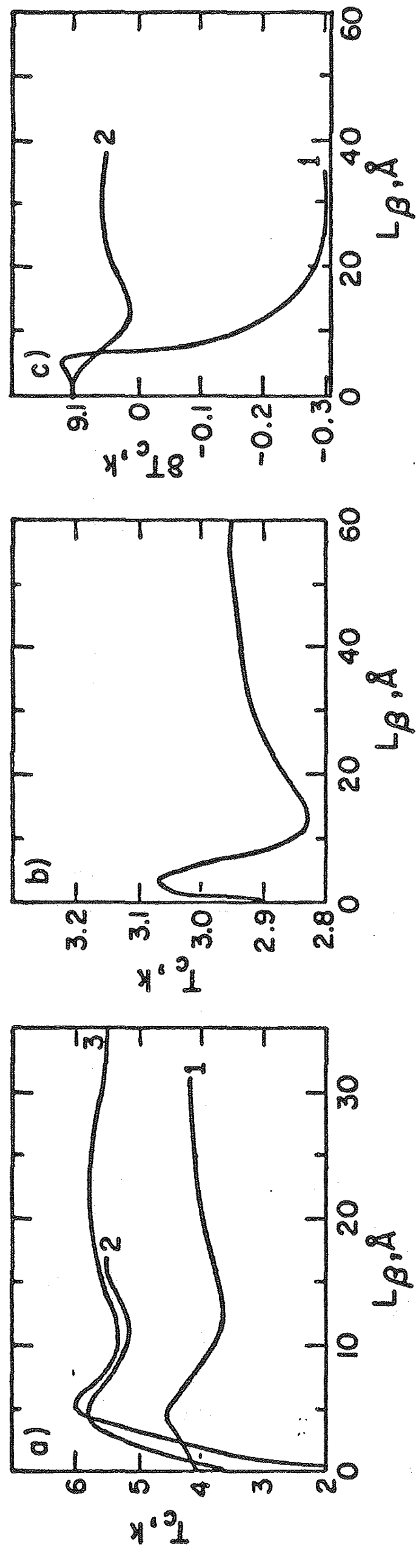

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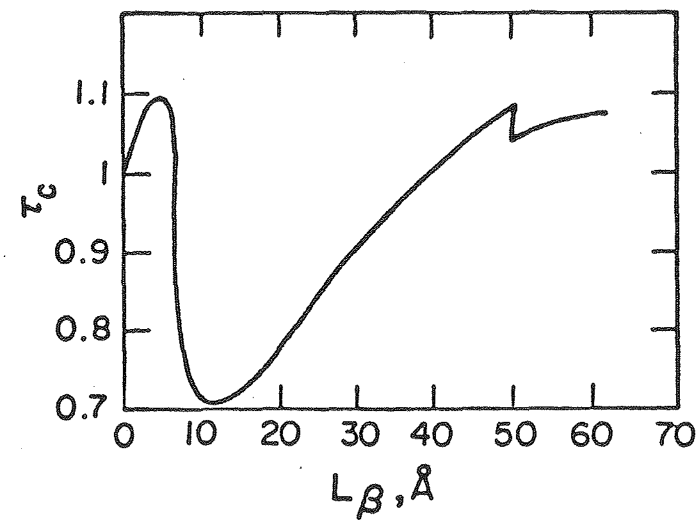

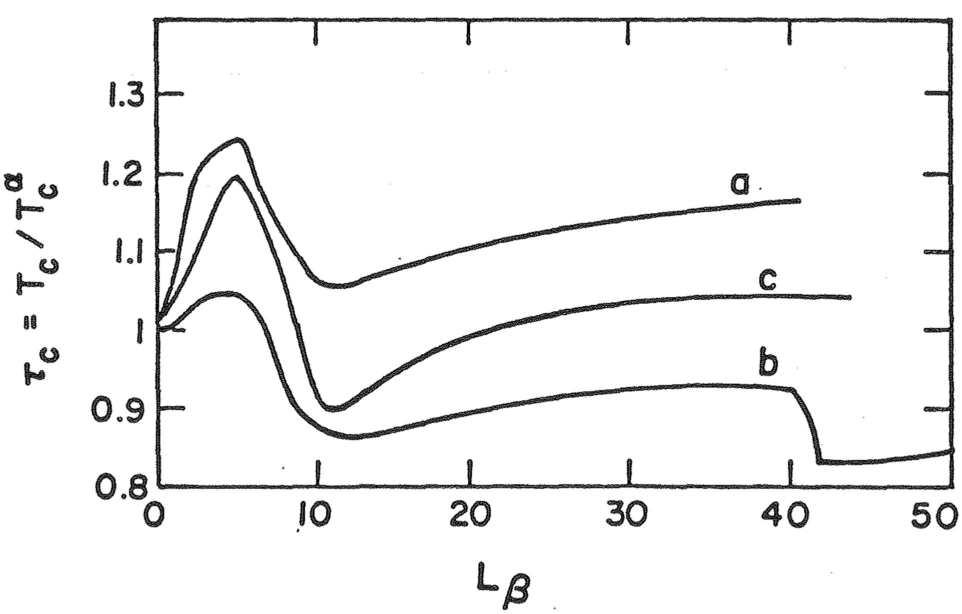

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